8-3 SOLVING QUADRATIC EQUATIONS BY COMPLETING THE SQUARE

Being able to recognize perfect-square trinomials can be helpful in solving quadratic equations. Recall that a perfect-square trinomial has two terms that are perfect squares. Its other term is twice the product of the square roots of the perfect-square terms.

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PERFECT-SQUARE TRINOMIALS

$x^2 +$	$2x$	$+ 1$	$=$	$(x + 1)^2$
$x^2 -$	$12x$	$+ 36$	$=$	$(x - 6)^2$
$x^2 +$	$5x$	$+ \frac{25}{4}$	$=$	$(x + \frac{5}{2})^2$
$x^2 -$	$2.4x$	$+ 1.44$	$=$	$(x - 1.2)^2$
$x^2 +$	$2ax$	$+ a^2$	$=$	$(x + a)^2$

twice the product of square roots of the perfect-square terms.

Note that in the examples above the constant (third) term of each trinomial is the square of one-half the coefficient of x in the second term.

EXAMPLE 1 Complete each member to show a perfect-square trinomial.

$$x^2 + 6x + ? = (x + ?)^2$$

SOLUTION The coefficient of x in the second term is 6. One-half the coefficient is 3. The square of 3 is 9. Thus, 9 is the constant term of the perfect-square trinomial.

$$x^2 + 6x + 9 = (x + 3)^2$$

To solve a quadratic equation by *completing the square*, collect in the left member the terms with the variable, and in the right member the constant terms. Complete a perfect-square for the left member and add the same number to the right member. Then find square roots of each member and complete any necessary steps that may remain.

EXAMPLE 2 Solve $x^2 + 6x = 27$ by completing the square.

SOLUTION $x^2 + 6x = 27$

Complete the perfect square for the left member and add the same number to the right member.

A_9
$$x^2 + 6x + 9 = 27 + 9$$
$$(x + 3)^2 = 36$$
$$\sqrt{(x + 3)^2} = \sqrt{36}$$
$$|x + 3| = 6$$

Therefore, $x + 3 = 6$, $x = 3$ or $x + 3 = -6$, $x = -9$.

CHECK

$x^2 + 6x = 27$	$x^2 + 6x = 27$
$3^2 + 6(3) \stackrel{?}{=} 27$	$(-9)^2 + 6(-9) \stackrel{?}{=} 27$
$27 = 27$ ✓	$27 = 27$ ✓

Completing the square is easiest when the coefficient of the second-degree term is 1. When it is different from 1, divide both members of the equation by that coefficient before completing the square.

EXAMPLE 3 Solve $-3y^2 + 5y = 2$ by completing the square.

SOLUTION
$$-3y^2 + 5y = 2$$

D_{-3}
$$y^2 - \tfrac{5}{3}y = -\tfrac{2}{3}$$
$$y^2 - \tfrac{5}{3}y + \left(\tfrac{5}{6}\right)^2 = -\tfrac{2}{3} + \tfrac{25}{36}$$
$$\left(y - \tfrac{5}{6}\right)^2 = \tfrac{1}{36}$$
$$\left|y - \tfrac{5}{6}\right| = \tfrac{1}{6}$$
$$y - \tfrac{5}{6} = \tfrac{1}{6} \quad \text{or} \quad y - \tfrac{5}{6} = -\tfrac{1}{6}$$
$$y = 1 \quad \Big| \quad y = \tfrac{2}{3}$$

CHECK The check is left for the student.

EXAMPLE 4 Solve $x^2 - 3x = 2$ by completing the square.

SOLUTION
$$x^2 - 3x = 2$$
$$x^2 - 3x + \left(\frac{3}{2}\right)^2 = 2 + \left(\frac{3}{2}\right)^2$$
$$\left(x - \frac{3}{2}\right)^2 = \frac{17}{4}$$
$$x - \frac{3}{2} = \pm\frac{\sqrt{17}}{2}$$
$$x = \frac{3 + \sqrt{17}}{2} \quad \text{or} \quad x = \frac{3 - \sqrt{17}}{2}$$

CHECK The check is left for the student.