III : $\boxed{f(x) = \frac{5x+6}{x+4}}$ $f'(x) = \frac{14}{(x+4)^2}$ $f \nearrow$.

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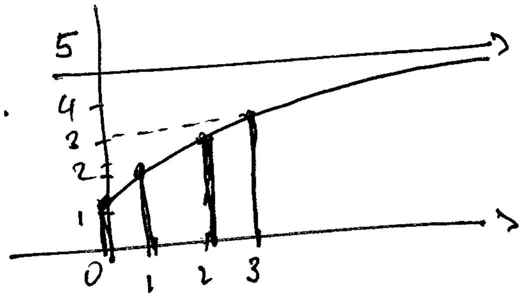
100 min.

x	0	$+\infty$
f	$\frac{3}{2}$	$\nearrow 5$



$a_0 = f(0) = \frac{3}{2} = 1{,}5$

$a_1 = f(1) = \frac{11}{5} \simeq 2{,}2$

$a_2 = f(2) = \frac{16}{6} = \frac{8}{3} \simeq 2{,}6$

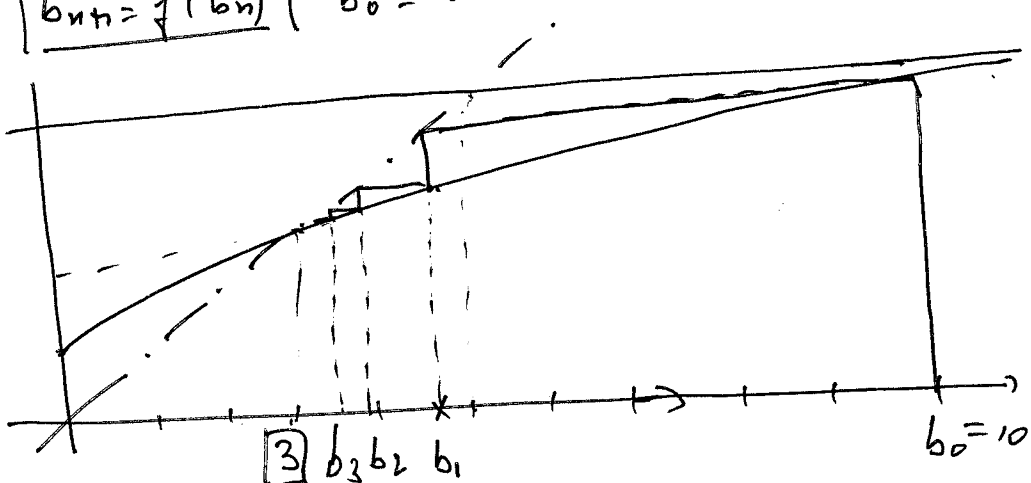
$a_3 = f(3) = \frac{21}{7} = 3$.

$\boxed{a_n = f(n)}$ $f \nearrow$ sur $[0\,;+\infty[$ donc $(a_n) \nearrow$.

$0 \leq \frac{3}{2} \leq a_n \leq 5$ car $\lim_{+\infty} f = 5$. (Rapport des termes de + haut degré).

$\lim(a_n) = \lim_{n \to +\infty} f(n) = \boxed{5}$.

$\boxed{b_{n+1} = f(b_n)}$ $b_0 = 10$.



$(b_n) \searrow$.

$3 \leq b_n \leq 10$.

$\lim b_n = 3$.

(Pt fixe de f)

$(\mathcal{S}_n)$ $\boxed{b_{n+1} \leq b_n}$. (i) (P_0) $b_1 \leq b_0$ $\boxed{\text{VRAI}}$ car $b_0 = 10$ $b_1 = f(10) = \frac{56}{14} = 4$

(ii) $(P_n) \overset{?}{\Longrightarrow} (P_{n+1})$ (n fixé)

$\Updownarrow$

$b_{n+1} \leq b_n \Longrightarrow f(b_{n+1}) \leq f(b_n) \Longrightarrow b_{n+2} \leq b_{n+1}$

car $f \nearrow$ sur $[0\,;10]$.

(iii) qq soit $n \in \mathbb{N}$, $\boxed{b_{n+1} \leq b_n}$. $\Longleftrightarrow (b_n) \searrow$.

(P_n) $0 \leq b_n \leq 10$ (i) (P_0) Vrai car $b_0 = 10$.

(ii) $(P_n) \overset{?}{\Longrightarrow} (P_{n+1})$.

$\Updownarrow$

$0 \leq b_n \leq 10 \overset{f \nearrow}{\Longrightarrow} f(0) \leq b_{n+1} \leq f(10) \Longrightarrow 0 \leq b_{n+1} \leq 10$

car $f(0) = \frac{3}{2} \geq 0$.

et $f(10) = 4 \leq 10$.

(iii) conclusion : (b_n) bornée par 0 et 10.

car qq soit $n \in \mathbb{N}$, $\boxed{0 \leq b_n \leq 10}$. (Principe d'induction)