- Etude d'une fonction trigonométrique.

$f(x) = \sin 2x - 2\cos x + 1$ $\left\{ \begin{array}{l} \sin 2x \to T_1 = \pi \\ \cos x \to T_2 = 2\pi \end{array} \right\} \Rightarrow \boxed{T = 2\pi}$.

période de f = PPCM des périodes de

intervalle d'étude longueur 2π : par exemple $[-\pi ; \pi]$ ou $\left[-\frac{\pi}{2} ; \frac{3\pi}{2}\right]$.

$f(x) = 1 \iff \sin 2x - 2\cos x + 1 = 1 \iff 2\sin x\cos x - 2\cos x = 0.$

$\iff 2\cos x[\sin x - 1] = 0 \iff \left\{ \begin{array}{l} \sin x = 1 \iff x = \frac{\pi}{2}\ [2\pi]. \\ \text{ou} \\ \cos x = 0 \iff \left\{ \begin{array}{l} x = \frac{\pi}{2}\ [2\pi] \\ \text{ou} \\ x = -\frac{\pi}{2}\ [2\pi]. \end{array} \right. \end{array} \right.$

Donc 2 sol : $\left\{ \frac{\pi}{2} ; -\frac{\pi}{2} \right\}$ (mod. 2π).

- $f\left(\frac{\pi}{2} - x\right) = \sin 2\left(\frac{\pi}{2} - x\right) - 2\cos\left(\frac{\pi}{2} - x\right) + 1 = \sin(\pi - 2x) - 2\sin x + 1.$

$= \sin 2x - 2\sin x + 1$ (car $\sin(\pi - \alpha) = \sin\alpha$).

- $f\left(\frac{\pi}{2} + x\right) = \sin 2\left(\frac{\pi}{2} + x\right) - 2\cos\left(\frac{\pi}{2} + x\right) = \sin(\pi + 2x) + 2\sin x + 1$

$= -\sin 2x + 2\sin x + 1.$ car $\sin(\pi + \alpha) = -\sin\alpha$.

- Donc $f\left(\frac{\pi}{2} + x\right) + f\left(\frac{\pi}{2} - x\right) = 2 \iff I\left(\frac{\pi}{2} ; 1\right)$ CENTRE de SYM. pour la courbe de f.

- $f'(x) = 2\cos 2x + 2\sin x = 2(1 - 2\sin^2 x) + 2\sin x.$

$= 2[-2\sin^2 x + \sin x + 1]. = 2(1 - \sin x)(1 + 2\sin x).$

pour $x \in [0 ; \pi]$.

on pose $X = \sin x$, $0 \leq X$

Sgn $f'(x)$ = Sgn $[-2X^2 + X + 1]$.

le trinôme $-2X^2 + X + 1$ a pour racines $\Delta = 9$

$\left\{ \begin{array}{l} X_1 = \frac{-1 + \sqrt{9}}{-4} = -\frac{1}{2} < 0. \\ \text{ou} \\ X_2 = \frac{-1 - \sqrt{9}}{-4} = 1. \end{array} \right.$

$X = 1 \iff \sin x = 1 \iff \boxed{x = \frac{\pi}{2}}$.

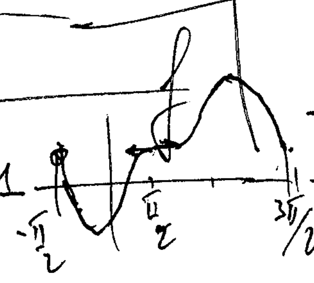
$X = -\frac{1}{2} \iff \sin x = -\frac{1}{2}$

$\iff x = \frac{7\pi}{6}$ ou $x = -\frac{5\pi}{6}$.

X	0		1
$-2X^2 + X + 1$		+	

x	0	$\frac{\pi}{2}$	π
$f'(x)$	+		+

x	$-\frac{\pi}{2}$		$-\frac{\pi}{6}$		0		$\frac{\pi}{2}$		π		$\frac{7\pi}{6}$		$\frac{3\pi}{2}$
$f'(x)$		−	0	+			+				0	−	
f	1	↘	$-\frac{3\sqrt{3}}{2}+1$	↗	−1	↗	1	↗	3	↗	$\frac{3\sqrt{3}}{2}+1$	↘	1



$f\left(\frac{7\pi}{6}\right) = \frac{\sqrt{3}}{2} + 2\frac{\sqrt{3}}{2} + 1 = \frac{3\sqrt{3}}{2} + 1 \simeq 3{,}5$

$f\left(-\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{2} - 2\frac{\sqrt{3}}{2} + 1 = -\frac{3\sqrt{3}}{2} + 1 \simeq -1{,}6$

$f'\left(\frac{\pi}{2}\right) = 0$